



Fig. 1. Lines of constant η on an r, z coordinate system.

Figure 1 shows lines of $\eta = \frac{v}{4\nu} \frac{r^2}{z} = \text{constant}$ superimposed over the original z, r coordinate system. The constants C_i that are listed follow the relation

$$C_1 > C_2 > C_3 > C_4 > C_5 \quad (3)$$

The intersection of lines of constant η with the line $r = R_0$ corresponds to η_0 , the cylinder boundary.

If, for example, we consider π_v , the appropriate boundary condition is

$$\pi_v = 1 \text{ at } \eta = \eta_0 \quad (4)$$

However, if π_v is a function of η alone, then π_v must equal 1 at $\eta = C_1, C_2, \dots, C_5$ since these lines cross the line $r = R_0$. The quadrant $r \geq 0, z \geq 0$ is, of course, densely filled with lines of constant η and all of these lines intersect $r = R_0$. Hence, the assumption that π_v is a function of η alone implies $\pi_v = 1$ everywhere. The above statement introduces a logical contradiction and one is forced to

conclude that π_v is not a function of η alone. Thus, the transformation leading to Equation (2) is incorrect and Equation (2) itself is not in proper form.

It should further be noted that the limiting value of C_f for large η should approach the value calculated for a moving continuous flat plate since the effects of curvature are not important in this region. According to Sakiadis (1)

$$C_f \text{ is proportional to } z^{-1/2} \quad (5)$$

whereas Vasudevan and Middleman's calculations seem to approach a functional form of

$$C_f \text{ is proportional to } z^{-1} \quad (6)$$

NOTATION

- C_f = local friction coefficient
- C_1, \dots, C_5 = constants
- D_0 = cylinder diameter, L
- f = function in Equation (2)
- r = radial coordinate, L
- R_0 = cylinder radius, L
- V = velocity of cylinder, L/t
- v_z = axial component of velocity, L/t
- z = axial coordinate, L

Greek Letters

- $\eta = \frac{Vr^2}{4\nu z}$ = similarity variable
- ν = kinematic viscosity, L^2/t
- Λ = constant in Equation (2)
- π = function in Equation (2)

LITERATURE CITED

1. Sakiadis, B. C., *AIChE J.*, 7, 221 (1961).
2. Vasudevan, G., and Stanley Middleman, *ibid.*, 16, 614 (1970).

Comment on the Communication of Fox and Hagin

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Our claim of a similarity transform for Equation (11) is in error. Inspection of Equation (14), in which f is defined as

$$f = -\gamma + \eta_0 + \frac{1}{\Lambda_i} + \int_{\eta_0}^{\eta} \Pi_v d\eta$$

indicates that for small η_0 , say $\eta_0 < 0.01$, f may be only weakly dependent on η_0 , especially near the jet surface. The good agreement between our theory and the work of Sakiadis in this range seems to bear this out.

The magnitude of the error in our results for $\eta_0 > 0.01$

is difficult to assess. As Fox and Hagin (see above) point out, and as is evident in our figures, our theory deviates significantly from Sakiadis' results for large η_0 . As we point out in Paragraph 1, page 618, of citation 2 above, Sakiadis states that his method does not work well for large η_0 . At the present time there is no accurate theory for this region.

It is fortunate that application of these results to fiber spinning will normally be such that $\eta_0 < 0.01$. Our application in jet stability studies has sometimes gone above $\eta_0 = 0.1$, but below $\eta_0 = 1$.

We wish to express our appreciation to Professor Cole of Clarkson College for his helpful comments on this matter.